

**Assignment -4 Class 12**  
**Relations and Functions**

1. choose the correct(✓) option:

(i) If a relation R on the set {1,2,3} be defined by  $R = \{(1,2)\}$ , then R is

- (a) reflexive            (b) transitive            (c) symmetric            (d) None of these

(ii) A relation R on R as  $aRb$  if  $a \geq b$ . Then R is

- (a) an equivalence relation            (b) reflexive, transitive but not symmetric  
(c) symmetric, transitive but not reflexive            (d) neither transitive nor reflexive but symmetric

(iii) Let  $f: R \rightarrow R$  be defined by  $f(x) = \frac{1}{x}$ ,  $\forall x \in R$  then f is

- (a) one one            (b) onto            (c) bijective            (d) f is not defined

(iv) Which of the following functions from Z to Z are bijections?

- (a)  $f(x) = x^3$             (b)  $f(x) = x + 2$             (c)  $f(x) = 2x + 1$             (d)  $f(x) = x^2 + 1$

(v) Let  $f: [2, \infty) \rightarrow R$  be the function defined by  $f(x) = x^2 - 4x + 5$ , then the range of f is

- (a) R            (b)  $[1, \infty)$             (c)  $[4, \infty)$             (d)  $[5, \infty)$

(vi) Let  $f: R \rightarrow R$  be defined by  $f(x) = \begin{cases} 2x & : x > 3 \\ x^2 & : 1 < x \leq 3 \\ 3x & : x \leq 1 \end{cases}$ , then  $f(-1) + f(2 + f(4))$  is

- (a) 9            (b) 14            (c) 5            (d) None of these

2. State true or false:

(i) Every function is invertible.

(ii) The compositions of functions is commutative.

(iii) Every relation which is symmetric and transitive is also reflexive.

(iv) Let:  $R \rightarrow R$  be the function defined by  $f(x) = \sin(3x+2) \forall x \in R$ , then f is invertible.

(v) Let  $f: R \rightarrow R$  be the bijection defined by  $f(x) = x^3 + 5$ . Then  $f^{-1}(x)$  is  $(x + 5)^{1/3}$ .

3. If  $f: R \rightarrow R$  is defined by  $f(x) = x^2 - 3x + 2$ , find  $f[f(x)]$ .

4. Functions  $f, g: R \rightarrow R$  are defined respectively by  $f(x) = x^2 + 3x + 1$  and  $g(x) = 2x - 3$ , find  $f \circ g$  and  $g \circ f$ .

5. Check whether the relation R on R (set of real numbers) defined by  $R = \{(a,b) : a \leq b^3\}$  is reflexive, symmetric or transitive.

6. If  $f: R \rightarrow R$  is given by  $f(x) = (3-x^3)^{1/3}$ , show that  $f \circ f = I_R$ , where  $I_R$  is the identity map on R.

7. Let  $f: N \rightarrow N$  be a function defined as  $f(x) = 4x^2 + 12x + 15$ . Show that  $f: N \rightarrow S$  is invertible (where S is the range of f). Find the inverse of f and hence, find  $f^{-1}(31)$  and  $f^{-1}(87)$ .

8. Consider  $f: R_+ \rightarrow [-9, \infty)$  given by  $f(x) = 5x^2 + 6x - 9$ , where  $R_+$  is the set of all non-negative real numbers. Prove that f is invertible with  $f^{-1}(y) = \frac{\sqrt{54+5y}-3}{5}$ .

9. If the function  $f: R \rightarrow R$  be defined by  $f(x) = 2x - 3$  and  $g: R \rightarrow R$  by  $g(x) = x^3 + 5$ , then find the value of  $(f \circ g)^{-1}(x)$ .

10. Let  $f: W \rightarrow W$  be defined as  $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$ .

Show that f is invertible and find the inverse of f. Here W is the set of whole numbers.

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